

Exercise 1.1 Solutions

Clean, step-by-step solved exercises formatted with beautiful styling, custom vector graphics, and standard mathematical proofs. Absolutely free of watermark and website markings.

Question 1

RATIONAL VS IRRATIONAL

Identify each of the following as a rational or irrational number:

✓ SOLUTIONS & ANALYSIS

(i) 2.353535

RATIONAL

It is a terminating decimal. Any terminating decimal is a rational number because it can be written as $\frac{2353535}{1000000}$.

(ii) $0.\overline{6}$

RATIONAL

It is a recurring (repeating) decimal: 0.6666... Repeating decimals can always be represented in the form $\frac{p}{q}$ (specifically, $\frac{2}{3}$).

(iii) 2.236067...

IRRATIONAL

It is a non-terminating, non-repeating decimal. The digits continue indefinitely without showing any periodic repetition.

(iv) $\sqrt{7}$

IRRATIONAL

The square root of a prime number (or any number which is not a perfect square) is always an irrational number.

(v) e

IRRATIONAL

Euler's number $e \approx 2.71828\dots$ is a transcendental mathematical constant that is non-terminating and non-repeating.

(vi) π

IRRATIONAL

$\pi \approx 3.14159\dots$ is a well-known transcendental constant which cannot be expressed exactly as a ratio of two integers.

(vii) $5 + \sqrt{11}$

IRRATIONAL

The sum of a rational number (5) and an irrational number ($\sqrt{11}$) is always an irrational number.

(viii) $\sqrt{3} + \sqrt{13}$

IRRATIONAL

The sum of two distinct square-root irrationals is also irrational because they do not cancel out.

(ix) $\frac{15}{4}$

RATIONAL

It is written explicitly in the form $\frac{p}{q}$ where both p and q are integers and $q \neq 0$.

(x) $(2 - \sqrt{2})(2 + \sqrt{2})$

RATIONAL

Simplifying: $2^2 - (\sqrt{2})^2 = 4 - 2 = 2$. Since 2 is an integer, it is a rational number ($\frac{2}{1}$).

Question 2

NUMBER LINE REPRESENTATION

Represent the following numbers on the number line:

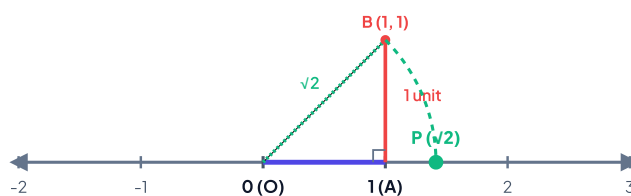
(i) $\sqrt{2}$

Geometric Construction:

1. Draw a standard number line. Let the origin be $O(0)$ and point A be at 1 unit. So, $OA = 1$ unit.
2. Construct a perpendicular segment $AB = 1$ unit at point A (so $AB \perp OA$).
3. Join O to B . By Pythagoras theorem, the hypotenuse OB is:

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ units}$$

4. Using a compass with center O and radius $OB = \sqrt{2}$, draw an arc that cuts the number line on the positive side at point P .
5. Point P represents the value $\sqrt{2} \approx 1.414$ on the number line.



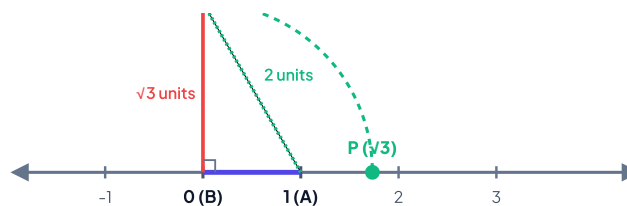
(ii) $\sqrt{3}$

Geometric Construction:

1. Let the segment $AB = 1$ unit on the number line from 0 to 1. Here, let B be at the origin 0, and A at 1.
2. At point $B(0)$, construct a perpendicular line segment.
3. From point $A(1)$, draw an arc of radius 2 units such that it intersects the perpendicular at point C .
4. Join A to C , creating a right-angled triangle ABC with base $AB = 1$ unit and hypotenuse $AC = 2$ units.
5. By Pythagoras theorem, the perpendicular side BC is:

$$BC = \sqrt{AC^2 - AB^2} = \sqrt{2^2 - 1^2} = \sqrt{3} \text{ units}$$

6. With $B(0)$ as center and radius $BC = \sqrt{3}$, draw an arc downwards to intersect the number line at point P .
7. Point P represents $\sqrt{3} \approx 1.732$ units from $B(0)$.



(iii) $4\frac{1}{3}$

Solution:

The fraction $4\frac{1}{3} = \frac{13}{3} \approx 4.333$ is a positive rational number that lies between 4 and 5.

1. Draw a number line focusing on the interval between 3, 4, and 5.
2. Divide the segment between 4 and 5 into 3 equal subparts.
3. Mark the first division point after 4. This point P represents the number $4\frac{1}{3}$.

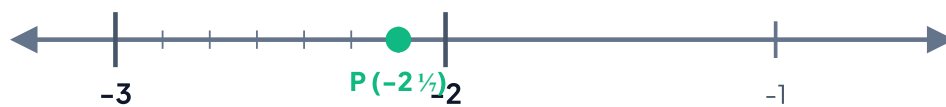


(iv) $-2\frac{1}{7}$

Solution:

The fraction $-2\frac{1}{7} = -\frac{15}{7} \approx -2.143$ is a negative rational number that lies between -2 and -3 .

1. Draw a number line focusing on the negative side, specifically between -1 , -2 , and -3 .
2. Divide the segment between -2 and -3 into 7 equal parts.
3. Since the value is negative, count 1 subdivision step to the left of -2 .
4. Mark point P . This point represents $-2\frac{1}{7}$.



(v) $\frac{5}{8}$

Solution:

The fraction $\frac{5}{8} = 0.625$ is a positive rational number that lies between 0 and 1.

1. Draw a number line focusing on the interval between 0 and 1.
2. Divide this interval into 8 equal parts.
3. Mark the 5th division point to the right of 0.
4. This point P represents the fraction $\frac{5}{8}$.



(vi) $2\frac{3}{4}$

Solution:

The fraction $2\frac{3}{4} = \frac{11}{4} = 2.75$ is a positive rational number that lies between 2 and 3.

1. Draw a number line focusing on the positive side, specifically between 1, 2, and 3.
2. Divide the segment between 2 and 3 into 4 equal parts.
3. Mark the 3th division point to the right of 2.
4. This point P represents the fraction $2\frac{3}{4}$.



Question 3

RECURRING TO P/Q FORM

Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i) $0.\overline{4}$

Let $x = 0.\overline{4}$

$$x = 0.4444... \quad \text{— (Equation 1)}$$

Since there is only one repeating digit in the period, multiply both sides of the equation by 10:

$$10x = 10 \times (0.4444...)$$

$$10x = 4.4444... \quad \text{— (Equation 2)}$$

Subtracting Equation 1 from Equation 2:

$$10x - x = (4.4444...) - (0.4444...)$$

$$9x = 4$$

$$x = \frac{4}{9}$$

Thus, $0.\overline{4} = \frac{4}{9}$

(ii) $0.\overline{37}$

Let $x = 0.\overline{37}$

$$x = 0.373737... \quad \text{— (Equation 1)}$$

Since there are two repeating digits in the period, multiply both sides of the equation by 100:

$$100x = 100 \times (0.373737...)$$

$$100x = 37.373737... \quad \text{— (Equation 2)}$$

Subtracting Equation 1 from Equation 2:

$$100x - x = (37.373737...) - (0.373737...)$$

$$99x = 37$$

$$x = \frac{37}{99}$$

Thus, $0.\overline{37} = \frac{37}{99}$

(iii) $0.\overline{21}$

Let $x = 0.\overline{21}$

$$x = 0.212121... \quad \text{--- (Equation 1)}$$

Since there are two repeating digits in the period, multiply both sides of the equation by 100:

$$100x = 100 \times (0.212121...)$$

$$100x = 21.212121... \quad \text{--- (Equation 2)}$$

Subtracting Equation 1 from Equation 2:

$$100x - x = (21.212121...) - (0.212121...)$$

$$99x = 21$$

$$x = \frac{21}{99}$$

Simplifying the fraction by dividing the numerator and denominator by their greatest common divisor (3):

$$x = \frac{21 \div 3}{99 \div 3} = \frac{7}{33}$$

Thus, $0.\overline{21} = \frac{7}{33}$

Question 4

PROPERTIES OF REAL NUMBERS

Name the property of real numbers used in the following statements:

Sr.	Statement	Property Used & Explanation
(i)	$(a + 4) + b = a + (4 + b)$	Associative Property w.r.t Addition Changing the grouping of the terms does not affect their sum.
(ii)	$\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$	Commutative Property w.r.t Addition Changing the order of the terms does not change their sum.

Sr.	Statement	Property Used & Explanation
(iii)	$x - x = 0$	Additive Inverse Property Adding a number to its negative (inverse) equals the additive identity (0).
(iv)	$a(b + c) = ab + ac$	Left Distributive Property of Multiplication over Addition Multiplication distributes over addition from the left side.
(v)	$16 + 0 = 16$	Additive Identity Property Adding zero to any number keeps the identity of the number unchanged.
(vi)	$100 \times 1 = 100$	Multiplicative Identity Property Multiplying any number by one keeps the identity of the number unchanged.
(vii)	$4 \times (5 \times 8) = (4 \times 5) \times 8$	Associative Property w.r.t Multiplication Changing the grouping of the factors does not affect their product.
(viii)	$ab = ba$	Commutative Property w.r.t Multiplication Changing the order of the factors does not change their product.

Question 5

INEQUALITY PROPERTIES

Name the property of inequalities used in the following statements:

Sr.	Statement	Inequality Property & Explanation
(i)	$-3 < -1 \Rightarrow 0 < 2$	Additive Property of Inequality Adding 3 to both sides maintains the inequality direction: $-3 + 3 < -1 + 3 \Rightarrow 0 < 2$.

Sr.	Statement	Inequality Property & Explanation
(ii)	If $a < b$ then $\frac{1}{a} > \frac{1}{b}$	Reciprocal Property of Inequality Taking the reciprocal of positive terms reverses the inequality direction (assuming $a, b > 0$).
(iii)	If $a < b$ then $a + c < b + c$	Additive Property of Inequality Adding any real number c to both sides does not change the inequality.
(iv)	If $ac < bc$ and $c > 0$ then $a < b$	Cancellation Property of Inequality w.r.t Multiplication Dividing both sides by a positive number c maintains the inequality.
(v)	If $ac < bc$ and $c < 0$ then $a > b$	Cancellation Property of Inequality w.r.t Multiplication Dividing both sides by a negative number c reverses the inequality direction.
(vi)	Either $a > b$ or $a = b$ or $a < b$	Trichotomy Property For any two real numbers a and b , exactly one of these three relations holds true.

Question 6

RATIONAL INSERTION

Insert two rational numbers between the given numbers:

(i) $\frac{1}{3}$ and $\frac{1}{4}$

Step 1: Find the first rational number by taking the average of the two numbers:

$$\text{Average } r_1 = \frac{\frac{1}{3} + \frac{1}{4}}{2}$$

$$\text{Average } r_1 = \frac{\frac{4+3}{12}}{2} = \frac{7}{12 \times 2} = \frac{7}{24}$$

Step 2: Find the second rational number by taking the average of $\frac{1}{3}$ and our new number $\frac{7}{24}$:

$$\text{Average } r_2 = \frac{\frac{1}{3} + \frac{7}{24}}{2}$$

$$\text{Average } r_2 = \frac{\frac{8+7}{24}}{2} = \frac{15}{24 \times 2} = \frac{15}{48}$$

Simplifying the fraction by dividing numerator and denominator by 3:

$$\text{Average } r_2 = \frac{5}{16}$$

Conclusion: Two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{5}{16}$.

(ii) 3 and 4

Step 1: Find the first rational number by taking the average of 3 and 4:

$$\text{Average } r_1 = \frac{3+4}{2} = \frac{7}{2} \quad (3.5)$$

Step 2: Find the second rational number by taking the average of $\frac{7}{2}$ and 4:

$$\text{Average } r_2 = \frac{\frac{7}{2} + 4}{2}$$

$$\text{Average } r_2 = \frac{\frac{7+8}{2}}{2} = \frac{15}{4} \quad (3.75)$$

Conclusion: Two rational numbers between 3 and 4 are $\frac{7}{2}$ and $\frac{15}{4}$.

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

Step 1: Find the first rational number by taking the average of the two fractions:

$$\text{Average } r_1 = \frac{\frac{3}{5} + \frac{4}{5}}{2}$$

$$\text{Average } r_1 = \frac{\frac{7}{5}}{2} = \frac{7}{10} \quad (0.7)$$

Step 2: Find the second rational number by taking the average of $\frac{3}{5}$ and $\frac{7}{10}$:

$$\text{Average } r_2 = \frac{\frac{3}{5} + \frac{7}{10}}{2}$$

$$\text{Average } r_2 = \frac{\frac{6+7}{10}}{2} = \frac{13}{20} \quad (0.65)$$

Note: You can also choose the average between $\frac{7}{10}$ and $\frac{4}{5}$, which is $\frac{3}{4}$ (0.75). Both are correct.

Conclusion: Two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{7}{10}$ and $\frac{13}{20}$.